Dirac notation

inner product	$\langle f \vert g \rangle = \int_{-\infty}^{\infty} f^*(x) g(x)\,\mathrm{d}x$	$\langle f \vert g \rangle = \langle g \vert f \rangle^*$
	$\Big\langle f \Big\vert \sum_i c_i g_i \Big\rangle = \sum_i c_i \langle f \vert g_i \rangle$	$\Big\langle \sum_i c_i g_i \Big\vert f \Big\rangle = \sum_i c_i^* \langle g_i \vert f \rangle$
Hermitian operator $\widehat{\mathrm{A}}$, Cauchy–Schwarz inequality	$\langle f \vert \widehat{\mathrm{A}} g \rangle = \langle \widehat{\mathrm{A}} f \vert g \rangle$	$\langle f \vert f \rangle \langle g \vert g \rangle \geq \big\vert \langle f \vert g \rangle \big\vert^2$
commutation relations	$\big[\widehat{\mathrm{A}}, \widehat{\mathrm{B}}\big] = \widehat{\mathrm{A}}\widehat{\mathrm{B}} - \widehat{\mathrm{B}}\widehat{\mathrm{A}}$	$\big[\widehat{\mathrm{x}}, \widehat{\mathrm{p}}_x\big] = \mathrm{i}\hbar.$
generalized Ehrenfest theorem, generalized uncertainty principle	$\dfrac{\mathrm{d}\langle A \rangle}{\mathrm{d}t} = \dfrac{1}{\mathrm{i}\hbar} \Big\langle \big[\widehat{\mathrm{A}}, \widehat{\mathrm{H}}\big] \Big\rangle$	$\Delta A\, \Delta B \geq \dfrac{1}{2} \Big\vert \Big\langle \big[\widehat{\mathrm{A}}, \widehat{\mathrm{B}}\big] \Big\rangle \Big\vert$

Orbital angular momentum

classical quantities	$L_z = x p_y - y p_x$	$L = I\omega$	$E_{\mathrm{rot}} = \dfrac{L^2}{2I}$	$\boldsymbol{\mu} = I\mathbf{A} = \gamma \mathbf{L}$
operators	$\widehat{\mathrm{L}}_z = -\mathrm{i}\hbar \left[x\dfrac{\partial}{\partial y} - y\dfrac{\partial}{\partial x} \right]$	$\widehat{\mathrm{L}}_z = -\mathrm{i}\hbar \dfrac{\partial}{\partial \phi}$	$\widehat{\mathrm{L}}^2 = \widehat{\mathrm{L}}_x^2 + \widehat{\mathrm{L}}_y^2 + \widehat{\mathrm{L}}_z^2$	
commutation relations	$\big[\widehat{\mathrm{L}}_x, \widehat{\mathrm{L}}_y\big] = \mathrm{i}\hbar\widehat{\mathrm{L}}_z$	$\big[\widehat{\mathrm{L}}_y, \widehat{\mathrm{L}}_z\big] = \mathrm{i}\hbar\widehat{\mathrm{L}}_x$	$\big[\widehat{\mathrm{L}}_z, \widehat{\mathrm{L}}_x\big] = \mathrm{i}\hbar\widehat{\mathrm{L}}_y$	$\big[\widehat{\mathrm{L}}^2, \widehat{\mathrm{L}}_z\big] = 0$
eigenvalues	$L^2 = l(l+1)\hbar^2$	$L_z = m\hbar$	$l = 0,\ 1,\ 2,\ \ldots,$	$m = 0,\ \pm 1,\ \ldots \pm l$

Spin angular momentum (spin-$\frac{1}{2}$)

general spin matrix, general eigenvectors	$\widehat{\mathrm{S}}_{\mathbf{n}} = \dfrac{\hbar}{2} \begin{bmatrix} \cos\theta & \mathrm{e}^{-\mathrm{i}\phi}\sin\theta \\ \mathrm{e}^{\mathrm{i}\phi}\sin\theta & -\cos\theta \end{bmatrix}$	$\vert \uparrow_{\mathbf{n}} \rangle = \begin{bmatrix} \cos(\theta/2) \\ \mathrm{e}^{\mathrm{i}\phi}\sin(\theta/2) \end{bmatrix}$	$\vert \downarrow_{\mathbf{n}} \rangle = \begin{bmatrix} -\mathrm{e}^{-\mathrm{i}\phi}\sin(\theta/2) \\ \cos(\theta/2) \end{bmatrix}$
spin matrices	$\widehat{\mathrm{S}}_x = \dfrac{\hbar}{2} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\widehat{\mathrm{S}}_y = \dfrac{\hbar}{2} \begin{bmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\widehat{\mathrm{S}}_z = \dfrac{\hbar}{2} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

commutation relations	$\big[\widehat{\mathrm{S}}_x, \widehat{\mathrm{S}}_y\big] = \mathrm{i}\hbar\widehat{\mathrm{S}}_z$	$\big[\widehat{\mathrm{S}}_y, \widehat{\mathrm{S}}_z\big] = \mathrm{i}\hbar\widehat{\mathrm{S}}_x$	$\big[\widehat{\mathrm{S}}_z, \widehat{\mathrm{S}}_x\big] = \mathrm{i}\hbar\widehat{\mathrm{S}}_y$	$\big[\widehat{\mathrm{S}}^2, \widehat{\mathrm{S}}_z\big] = 0$
eigenvalues	$S^2 = s(s+1)\hbar^2$	$S_z = m_s\hbar$	$s = \frac{1}{2}, \quad m_s = \pm\frac{1}{2}$	
energy levels	$E_{\mathrm{mag}} = -\boldsymbol{\mu} \cdot \mathbf{B}$	$\boldsymbol{\mu} = \gamma_s \mathbf{S}$	$\widehat{\mathrm{H}} = -\gamma_{\mathrm{s}} B \widehat{\mathrm{S}}_{\mathbf{n}}$	

Identical particles

singlet spin ket		$\frac{1}{\sqrt{2}}\big(\vert\uparrow\downarrow\rangle - \vert\downarrow\uparrow\rangle\big) = \vert 0, 0\rangle$	
triplet spin kets	$\vert\uparrow\uparrow\rangle = \vert 1, 1\rangle$	$\frac{1}{\sqrt{2}}\big(\vert\uparrow\downarrow\rangle + \vert\downarrow\uparrow\rangle\big) = \vert 1, 0\rangle$	$\vert\downarrow\downarrow\rangle = \vert 1, -1\rangle$

	spin	total wave function	exclusion principle	composite particle
fermion	$s = \frac{1}{2}, \frac{3}{2}, \ldots$	antisymmetric	yes	odd number of fermions
boson	$s = 0, 1, 2, \ldots$	symmetric	no	even number of fermions

Complex numbers

$z = x + \mathrm{i}y = r\mathrm{e}^{\mathrm{i}\theta}$	$z^* = x - \mathrm{i}y = r\mathrm{e}^{-\mathrm{i}\theta}$	$\lvert z\rvert^2 = zz^* = x^2 + y^2 = r^2$
$\mathrm{Re}(z) = \dfrac{z + z^*}{2}$	$\mathrm{Im}(z) = \dfrac{z - z^*}{2\mathrm{i}}$	$z^n = r^n \mathrm{e}^{\mathrm{i}n\theta}$
$\mathrm{e}^{\mathrm{i}\theta} = \cos\theta + \mathrm{i}\sin\theta$	$\cos\theta = \dfrac{\mathrm{e}^{\mathrm{i}\theta} + \mathrm{e}^{-\mathrm{i}\theta}}{2}$	$\sin\theta = \dfrac{\mathrm{e}^{\mathrm{i}\theta} - \mathrm{e}^{-\mathrm{i}\theta}}{2\mathrm{i}}$
$\mathrm{e}^{\pm\mathrm{i}\pi} = -1$	$\mathrm{e}^{\mathrm{i}\pi/2} = \mathrm{i}$	$\mathrm{e}^{-\mathrm{i}\pi/2} = -\mathrm{i}$

Elementary functions $(a > 0, b > 0)$

$\mathrm{e}^x\mathrm{e}^y = \mathrm{e}^{x+y}$	$\ln a + \ln b = \ln(ab)$	$\mathrm{e}^{\ln a} = \ln(\mathrm{e}^a) = a$
$\mathrm{e}^x = \cosh x + \sinh x$	$\cosh x = \dfrac{\mathrm{e}^x + \mathrm{e}^{-x}}{2}$	$\sinh x = \dfrac{\mathrm{e}^x - \mathrm{e}^{-x}}{2}$
$\cos(\theta \pm \pi) = -\cos\theta$	$\sin(\theta \pm \pi) = -\sin\theta$	$\tan(\theta \pm \pi) = \tan\theta$
$\cos(\theta + \pi/2) = -\sin\theta$	$\sin(\theta + \pi/2) = \cos\theta$	$\tan(\theta + \pi/2) = -\cot\theta$
$\cos(\theta - \pi/2) = \sin\theta$	$\sin(\theta - \pi/2) = -\cos\theta$	$\tan(\theta - \pi/2) = -\cot\theta$
$\cos(2\theta) = \cos^2\theta - \sin^2\theta$	$\sin(2\theta) = 2\sin\theta\cos\theta$	$\tan(2\theta) = 2\tan\theta/(1 - \tan^2\theta)$

$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$	$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$
$\sin A\sin B = \frac{1}{2}\big(\cos(A - B) - \cos(A + B)\big)$	$\cos A\cos B = \frac{1}{2}\big(\cos(A - B) + \cos(A + B)\big)$
$\sin A\cos B = \frac{1}{2}\big(\sin(A - B) + \sin(A + B)\big)$	$\cos^2 A + \sin^2 A = 1$

Physical constants

Planck's constant	h	$6.63 \times 10^{-34}\,\mathrm{J\,s}$	Planck's constant/2π	$\hbar$	$1.06 \times 10^{-34}\,\mathrm{J\,s}$
vacuum speed of light	c	$3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$	Coulomb law constant	$\frac{1}{4\pi\varepsilon_0}$	$8.99 \times 10^{9}\,\mathrm{m\,F^{-1}}$
permittivity of free space	ε_0	$8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$	permeability of free space	μ_0	$4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}}$
Boltzmann's constant	k	$1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$	Avogadro's constant	N_m	$6.02 \times 10^{23}\,\mathrm{mol^{-1}}$
electron charge	$-e$	$-1.60 \times 10^{-19}\,\mathrm{C}$	proton charge	e	$1.60 \times 10^{-19}\,\mathrm{C}$
electron mass	m_e	$9.11 \times 10^{-31}\,\mathrm{kg}$	proton mass	m_p	$1.67 \times 10^{-27}\,\mathrm{kg}$
Bohr radius	a_0	$5.29 \times 10^{-11}\,\mathrm{m}$	atomic mass unit	u	$1.66 \times 10^{-27}\,\mathrm{kg}$

Definite integrals for positive integers n and m

$$\int_{-a}^{a} f(x)\,\mathrm{d}x = 0 \qquad (f(x) \text{ an odd function})$$

$$\int_{-a}^{a} f(x)\,\mathrm{d}x = 2\int_{0}^{a} f(x)\,\mathrm{d}x \qquad (f(x) \text{ an even function})$$

$$\int_{0}^{\pi} \sin(nx)\sin(mx)\,\mathrm{d}x = \frac{\pi}{2}\,\delta_{nm}$$

$$\int_{0}^{\pi} \cos(nx)\cos(mx)\,\mathrm{d}x = \frac{\pi}{2}\,\delta_{nm}$$

$$\int_{-\pi/2}^{\pi/2} \sin(nx)\sin(mx)\,\mathrm{d}x = \frac{\pi}{2}\,\delta_{nm} \qquad (n+m \text{ even})$$

$$\int_{-\pi/2}^{\pi/2} \cos(nx)\cos(mx)\,\mathrm{d}x = \frac{\pi}{2}\,\delta_{nm} \qquad (n+m \text{ even})$$

$$\int_{0}^{n\pi} \cos^2 x\,\mathrm{d}x = \frac{n\pi}{2}$$

$$\int_{0}^{n\pi} \sin^2 x\,\mathrm{d}x = \frac{n\pi}{2}$$

$$\int_{0}^{n\pi} x\cos^2 x\,\mathrm{d}x = \frac{n^2\pi^2}{4}$$

$$\int_{0}^{n\pi} x\sin^2 x\,\mathrm{d}x = \frac{n^2\pi^2}{4}$$

$$\int_{0}^{n\pi} x^2\cos^2 x\,\mathrm{d}x = \frac{n^3\pi^3}{6} + \frac{n\pi}{4}$$

$$\int_{0}^{n\pi} x^2\sin^2 x\,\mathrm{d}x = \frac{n^3\pi^3}{6} - \frac{n\pi}{4}$$

$$\int_{-n\pi/2}^{n\pi/2} \cos^2 x\,\mathrm{d}x = \frac{n\pi}{2}$$

$$\int_{-n\pi/2}^{n\pi/2} \sin^2 x\,\mathrm{d}x = \frac{n\pi}{2}$$

$$\int_{-n\pi/2}^{n\pi/2} x^2\cos^2 x\,\mathrm{d}x = \frac{n^3\pi^3}{24} + \frac{n\pi}{4}(-1)^n$$

$$\int_{-n\pi/2}^{n\pi/2} x^2\sin^2 x\,\mathrm{d}x = \frac{n^3\pi^3}{24} - \frac{n\pi}{4}(-1)^n$$

$$\int_{-n\pi/2}^{n\pi/2} x^2\cos x\,\mathrm{d}x = (-1)^{(n+3)/2}\left(\frac{n^2\pi^2}{2} - 4\right), \ (n \text{ odd})$$

$$\int_{-n\pi/2}^{n\pi/2} x^2\cos x\,\mathrm{d}x = (-1)^{n/2}\ 2\pi n, \qquad (n \text{ even})$$

For $a > 0,\ n \geq 0,\ m \geq 1$:

$$\int_{-\infty}^{\infty} \mathrm{e}^{-x^2/a^2}\mathrm{d}x = a\sqrt{\pi}$$

$$\int_{0}^{\infty} x^n\mathrm{e}^{-x}\mathrm{d}x = n!$$

$$\int_{0}^{\infty} x^{2n+1}\mathrm{e}^{-x^2/a^2}\mathrm{d}x = \frac{n!}{2}a^{2n+2} \qquad (n \geq 0)$$

$$\int_{-\infty}^{\infty} \mathrm{e}^{-x^2}\mathrm{e}^{\mathrm{i}kx}\,\mathrm{d}x = \sqrt{\pi}\,\mathrm{e}^{-k^2/4} \qquad (k \text{ real})$$

$$\int_{-\infty}^{\infty} x^{2n}\mathrm{e}^{-x^2/a^2}\mathrm{d}x = \frac{1\times 3\times\cdots\times(2n-1)}{2^n}\,a^{2n+1}\sqrt{\pi}, \quad \text{for } (n \geq 1)$$

$$n! = 1\times 2\times\cdots\times n \qquad 0! = 1$$